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# ERROR ANALYSIS FOR A LAMINAR FLOW FLUID VISCOMETER

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# ERROR ANALYSIS FOR A LAMINAR FLOW FLUID VISCOMETER

# ANALYSE D'ERREUR POUR UN VISCOSIMÈTRE POUR FLUIDES À ÉCOULEMENT LAMINAIRE

W.F. Hayes
H.G. Tucker

Technical Report

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## ABSTRACT

A continuous reading fluid viscometer based on two-dimensional laminar flow, has been patented and the associated prototype sensor hardware tested. Commercially available continuous reading viscometers, which rely upon the detection of a viscosity-dependent pressure drop along a fluid flow passage,typically require either a constant or an accurately known fluid flow rate. This viscometer differs from such commercial viscometers in that the fluid viscosity may be derived directly from wall static pressure measurements in suitably shaped laminar flow passages, provided that the fluid density is either known or separately measured. The commercial potential of the patented viscometer is strongly dependent upon the viscometer precision attainable in practice as limited by the errors in pressure measurement and fluid density estimation.

Practical guidelines are suggested for the design of the laminar flow passages of the viscometer such that accurate theoretical prediction of viscometer performance is possible.

This paper assesses, using derived algorithms which model the viscometer fluid laminar flow process, the effects of errors in pressure measurement and density estimation upon the precision to which the fluid viscosity may be predicted. It is shown that the viscosity prediction error may be significantly reduced by the appropriate optimization of the geometry of the laminar flow passages for any specific fluid flow rate and viscosity operational range. For measurement instruments, as exemplified by the patented laminar flow viscometer, such an error analysis technique provides for the prediction of the instrument operational performance and indicates the sensitivity of the resultant precision to the various contributing error sources.



## RÉSUMÉ

Un viscosimètre pour fluides à lecture continue, basé sur l'écoulement laminaire bidimensionnel, a été breveté et testé. Les viscosimètres à lecture continue actuellement disponibles dans le commerce, qui sont basés sur la mesure d'une diminution de la pression dépendant de la viscosité le long d'un passage destiné à d'écoulement du fluide, exigent un débit constant ou connu avec précision. L'appareil dont il est ici question diffère des viscosimètres classiques par le fait qu'il permet d'obtenir la directement la viscosité à partir de mesures de la pression statique à la paroi d'un passage pour écoulement laminaire de forme convenable, pourvu que la densité du fluide soit connue ou mesurée séparément. Les possibilités d'exploitation commerciale du viscosimètre breveté sont intimement liées à la précision des mesures de la viscosité que permet en pratique l'appareil, elle même limitée par les erreurs de mesure de la pression et d'estimation de la densité du fluide.

On propose des principes directives pratiques pour la conception des passages pour écoulement laminaire du viscosimètre qui permettront une prévision théorique précise du rendement de l'appareil.

Dans la présente étude on évalue, au moyen d'algorithmes de modélisation du processus de l'écoulement laminaire du fluide dans le viscosimètre, les effets d'erreurs de mesure de la pression et d'estimation de la densité sur la précision de la prévision de la viscosité du fluide. On démontre que l'erreur de prévision de la viscosité peut être sensiblement réduite par une optimisation convenable de la géométrie des passages destinés à l'écoulement laminaire pour tout débit spécifique de fluide et toute plage opérationnelle de viscosités. Pour les instruments de mesure, comme l'illustre le cas du viscosimètre à écoulement laminaire breveté, une telle méthode d'analyse d'erreur permet de prédire le rendement opérationnel de l'instrument et indique la sensibilité de la précision résultante aux diverses sources d'erreur.

## CONTENTS


| | | Page |
|---|---|---|
| | ABSTRACT/RÉSUMÉ | (i) |
| | LIST OF ILLUSTRATIONS | (iii) |
| | LIST OF SYMBOLS | (iv) |
| 1.0 | INTRODUCTION | 1 |
| 2.0 | DERIVATION OF FLUID VISCOSITY | 1 |
| 3.0 | ASSESSMENT OF NON-DIMENSIONALIZED VISCOSITY ERROR STANDARD DEVIATION | 6 |
| 4.0 | PRACTICAL CONSIDERATIONS IN PASSAGE DESIGN | 9 |
| 5.0 | VISCOMETER ERROR ASSESSMENT FOR TWO POSSIBLE DESIGN CONFIGURATIONS | 11 |
| | 5.1 Single Pressure Transducer Viscometer Design | 11 |
| | 5.2 Dual Pressure Transducer Viscometer Design | 17 |
| 6.0 | CONCLUSIONS | 24 |
| 7.0 | REFERENCES | 25 |
| | APPENDIX A: Derivation of Non-Dimensionalized Viscosity Error Standard Deviation | A1 |


## LIST OF ILLUSTRATIONS


| | | |
|---|---|---|
| 1. | Definition of Viscometer Laminar Flow Parallel Passage Geometry (not to scale). | 3 |
| 2. | Definition of Viscometer Laminar Flow Convergent Passage Geometry (not to scale). | 4 |
| 3. | Schematic of Viscometer System Using Single Pressure Transducer. | 7 |
| 4. | Characteristics of the Normal or Gaussian Error Distribution. | 8 |
| 5. | Configuration of "Baseline Design" Experimental Viscometer Showing Laminar Flow Passage Geometries. | 13 |
| 6. | Viscometer Error Dependency upon Geometry of Laminar Flow Passages (single pressure transducer viscometer configuration). | 14 |

## LIST OF ILLUSTRATIONS (cont'd)


| Figure | | Page |
|---|---|---|
| 7 | Disassembled "Baseline Design" Viscometer Flow Element. | 19 |
| 8 | Experimental Flow Test Rig for Viscometer Flow Element. | 20 |
| 9 | Variation of Viscosity Error with Deviation of Fluid Flow Rate or Viscosity from Design Point (single pressure transducer viscometer configuration). | 21 |
| 10 | Variation of Viscosity Error and Flow Passage Pressure Drops with Convergent Passage Height (dual pressure transducer viscometer configuration). | 22 |
| 11 | Variation of Viscosity Error and Flow Passage Pressure Drops with Convergent Passage Radius Ratio (dual pressure transducer viscometer configuration). | 23 |


## LIST OF SYMBOLS

| Symbol | | Units |
|---|---|---|
| A, B | constants dependent upon fluid density and flow passage geometry | |
| G | fluid mass flow rate | kg $s^{-1}$ |
| h | flow passage height | m |
| $h_c$ | height of convergent flow passage | m |
| $h_p$ | height of parallel flow passage | m |
| $K_{CI}$ | convergent passage inertia pressure drop coefficient | $m^{-4}$ |
| $K_{CV}$ | convergent passage viscous pressure drop coefficient | $m^{-3}$ |
| $K_p$ | parallel passage viscous pressure drop coefficient | $m^{-3}$ |
| $K_R$ | $\frac{K_{CV}}{K_p}$ = ratio of coefficient of viscous pressure drop in the convergent to that in the parallel flow passage | |
| $l_e$ | flow passage entrance length | m |
| $l_p$ | length of parallel passage between static pressure taps | m |
| $\Delta P_1$, $\Delta P_2$ | static pressure drops across passages 1 and 2 respectively | N $m^{-2}$ |
| $\Delta P$ | measured static pressure drop | N $m^{-2}$ |

## LIST OF SYMBOLS (cont'd)

| Symbol | | Unit |
|---|---|---|
| $\Delta P_C$ | measured static pressure drop across convergent flow passage | (N $m^{-2}$) |
| $\Delta P_{CI}$ | fluid inertia-induced portion of static pressure drop across convergent flow passage | (N $m^{-2}$) |
| $\Delta P_{CV}$ | fluid viscosity-induced portion of static pressure drop across convergent flow passage | (N $m^{-2}$) |
| $\Delta P_P$ | measured static pressure drop across parallel flow passage | (N $m^{-2}$) |
| $P_e$ | maximum error in pressure measurement | (N $m^{-2}$) |
| $P_{ec}$ | maximum error in convergent flow passage pressure measurement | (N $m^{-2}$) |
| $P_{ep}$ | maximum error in parallel flow passage pressure measurement | (N $m^{-2}$) |
| $R_e$ | $\frac{\rho Vh}{\mu}$ = maximum Reynolds Number in flow passage | |
| $r_1$ | radius to upstream pressure tap in convergent flow passage | (m) |
| $r_2$ | radius to downstream pressure tap in convergent flow passage | (m) |
| V | fluid velocity averaged across the minimum flow passage cross-section | (m $s^{-1}$) |
| $w_p$ | width of parallel passage | (m) |
| $\mu$ | Newtonian fluid viscosity | (N s $m^{-2}$) |
| $\mu_e$ | error in viscometer prediction of fluid viscosity | (N s $m^{-2}$) |
| $\rho$ | fluid density | (kg $m^{-3}$) |
| $\rho_e$ | maximum error in fluid density estimation | (kg $m^{-3}$) |
| $\sigma$ | standard deviation of non-dimensionalized viscosity prediction error | |
| $\sigma_{ST}$ | standard deviation of non-dimensionalized viscosity prediction error for single pressure transducer viscometer configuration | |

# ERROR ANALYSIS FOR A LAMINAR FLOW FLUID VISCOMETER

## 1.0 INTRODUCTION

A patented continuous reading Newtonian fluid viscometer based on two-dimensional laminar flow (Hayes et al., 1976 and Hayes et al., 1977) may be used to predict fluid viscosity from wall static pressure measurements, provided the fluid density is known. This patented apparatus eliminates the need to provide a constant and accurately known fluid flow rate through a suitable fluid flow passage in order to generate a predictable viscosity-dependent pressure drop as is typical with conventional commercially available continuous reading viscometers. The commercial viability of this concept is significantly dependent upon the viscometer precision attainable as limited by errors in pressure measurement and fluid density estimation. Accordingly, it is appropriate to assess the interdependency of the passage geometry and fluid flow rate, as well as the errors associated with the fluid density estimation and fluid pressure measurement upon the uncertainty in the predicted fluid viscosity. The analysis of the interdependency of these effects provides a means for optimizing, within practical limitations, the design of the viscometer so as to satisfy specific operational requirements.

## 2.0 DERIVATION OF FLUID VISCOSITY

The viscosity of any Newtonian fluid may be explicitly determined using two series-connected fluid flow devices such that the pressure drop '$\Delta P_1$' in one passage is solely fluid viscous shear induced and the pressure drop '$\Delta P_2$' in the other passage is solely fluid inertia induced. Herein:

$$\Delta P_1 = A\ \mu\ G \tag{1}$$

and

$$\Delta P_2 = B\ G^2 \tag{2}$$

Since the fluid mass flow rate 'G' will be common to the two series-connected passages, Eqn. (1) and (2) may be readily solved simultaneously for the fluid viscosity '$\mu$' by eliminating the mass flow rate 'G'.

A viscosity-dependent pressure drop, as per Eqn. (1), may be readily hardware implemented by means of a thin laminar flow two-dimensional parallel passage as shown schematically in Fig. 1. It can be readily shown from fundamental fluid dynamic considerations (Schlichting, H., 1968, Chap. 5) that the static pressure drop across such a laminar flow passage is given by:

$$\Delta P_p = \frac{12\ l_p\ \mu\ G}{\rho\ w_p\ (h_p)^3}$$

$$= \frac{K_p\ \mu\ G}{\rho} \qquad (N\ m^{-2}) \tag{3}$$

where the passage geometry parameters $l_p$, $w_p$ and $h_p$ are as per Fig. 1 and where '$K_p$' is referred to as the parallel passage viscous pressure drop coefficient and is defined as:

$$K_p = \frac{12\ l_p}{w_p\ h_p^3} \quad (m^{-3}) \tag{4}$$

An inertia induced pressure drop, as per Eqn. (2) may only be hardware implemented by means of a turbulent flow device such as an orifice plate. However, turbulent flow devices suffer from the following deficiencies:

- turbulent flow inherently implies poor signal-to-noise ratio pressure measurement which would adversely affect the overall accuracy of viscosity determination.

- the performance of such turbulent flow devices is not amenable to theoretical modelling and thus the constant 'B' must be empirically determined by extensive and precise calibration.

- turbulent flow tends to physically wear the fluid flow passage, particularly across the high fluid velocity orifice, causing long-term calibration drift.

Thus, it is advantageous in practice to avoid the use of a turbulent flow device and to use a laminar flow passage in which the pressure drop is a function of both fluid viscous effects and fluid inertia effects. Hardware implementation of such a device in the case of the patented viscometer, is by means of a thin radial two-dimensional inflow fluid flow passage as shown in Fig. 2. It can be shown from fundamental fluid dynamic considerations that the static pressure drop across such a convergent flow passage is given, as per Eqn. A-32 of Hayes, W.F. and Tucker, H.G., 1974, by:

$$\begin{aligned} \Delta P_C &= \Delta P_{Cv} + \Delta P_{CI} \qquad (N\ m^{-2}) \\ &= \frac{K_{CV}\ \mu\ G}{\rho} + \frac{K_{CI}\ G^2}{\rho} \\ &= \frac{6\ \mu\ G}{\rho\pi(h_c)^3}\left|\ln\ \frac{r_2}{r_1}\right| + \frac{0.605\ G^2}{\rho(2\pi h_c)^2}\left|\frac{1}{r_2^2} - \frac{1}{r_1^2}\right| \end{aligned} \tag{5}$$

where the passage geometrical parameters $r_1$, $r_2$ and $h_c$ are as per Fig. 2 and where '$K_{CV}$' is referred to as the convergent passage viscous pressure drop coefficient and is defined as:

STATIC PRESSURE DROP BETWEEN TAPS =

$$\Delta P_p = \frac{K_p \mu G}{\rho}$$

WHERE G = MASS FLOW RATE

$\rho$ = FLUID DENSITY

$\mu$ = FLUID VISCOSITY

$$K_p = \frac{12 \ell_p}{w_p h_p^3}$$

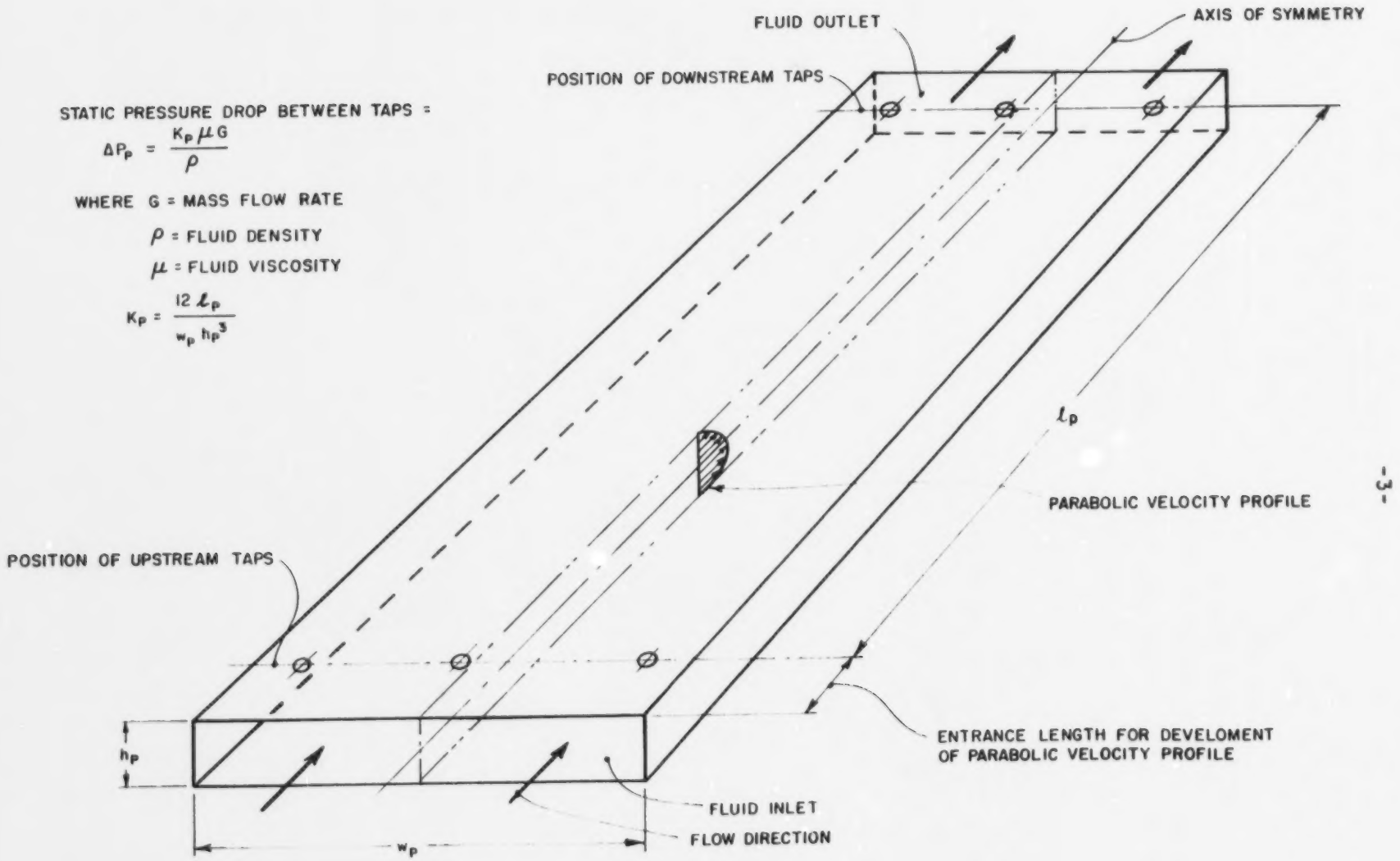



FIGURE 1

DEFINITION OF VISCOMETER LAMINAR FLOW PARALLEL PASSAGE GEOMETRY (NOT TO SCALE)

STATIC PRESSURE DROP BETWEEN TAPS =

$$\Delta P_c = \frac{K_{cv}\mu G}{\rho} + \frac{K_{c1}G^2}{\rho}$$

WHERE G = MASS FLOW RATE

$\rho$ = FLUID DENSITY

$\mu$ = FLUID VISCOSITY

$$K_{cv} = \frac{6}{\pi h_c^3}\left|\ln\left(\frac{r_2}{r_1}\right)\right|$$

$$K_{c1} = \frac{0.605}{(2\pi h_c)^2}\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right]$$

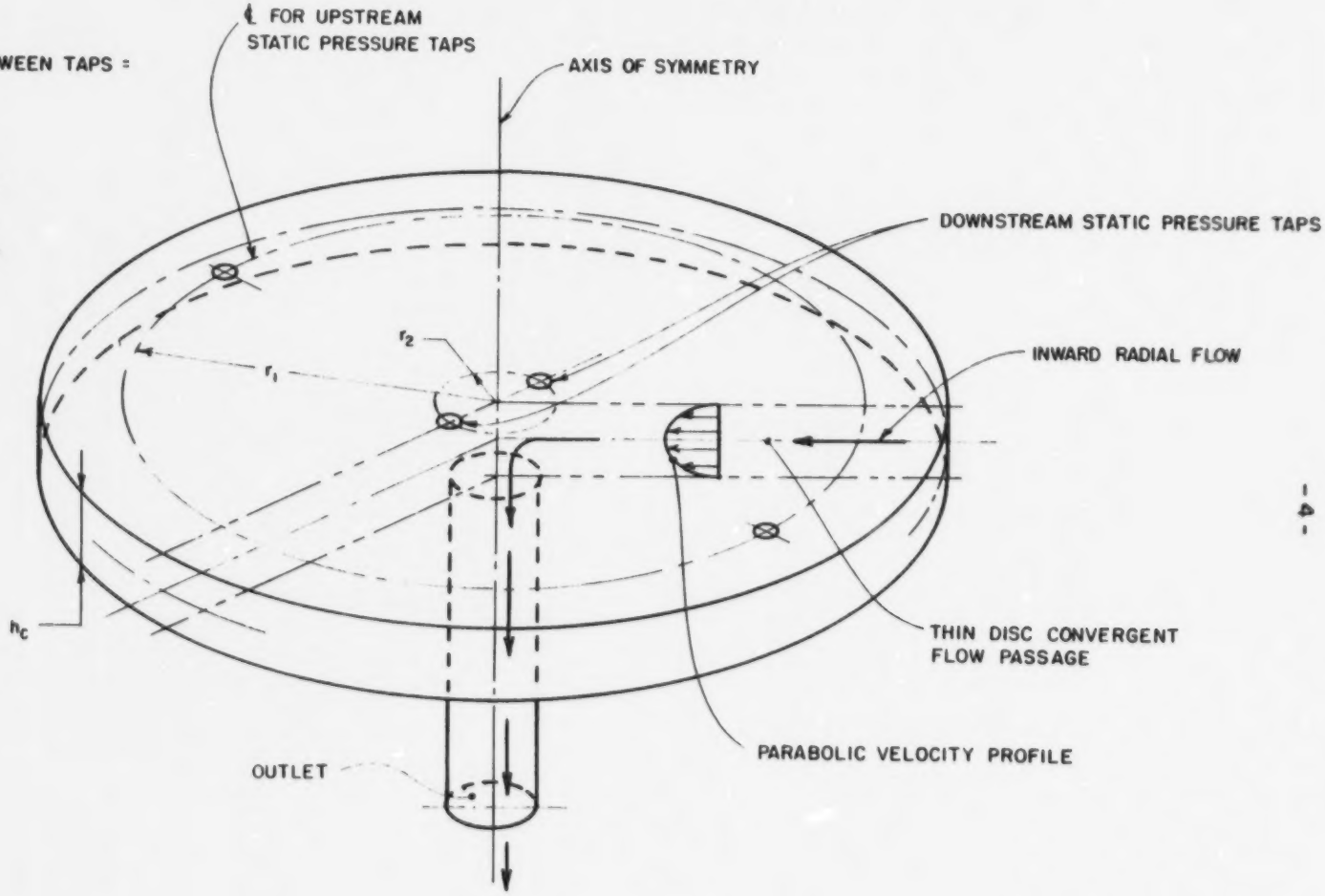



FIGURE 2

DEFINITION OF VISCOMETER LAMINAR FLOW CONVERGENT PASSAGE GEOMETRY (NOT TO SCALE)

$$K_{CV} = \frac{6}{\pi(h_c)^3} \left| \ln \frac{r_2}{r_1} \right| \ (m^{-3}) \qquad (6)$$

and where '$K_{CI}$' is referred to as the convergent passage inertia pressure drop coefficient and is defined as:

$$K_{CI} = \frac{0.605}{(2\pi h_c)^2} \left| \frac{1}{r_2^2} - \frac{1}{r_1^2} \right| (m^{-4}) \qquad (7)$$

Equations (3) and (5) may be readily solved simultaneously for the fluid viscosity '$\mu$' wherein:

$$\mu = \frac{\sqrt{K_{CI}}}{K_P} \frac{\Delta P_P \sqrt{\rho}}{\sqrt{\Delta P_C - K_R \Delta P_P}}$$

$$= \frac{\sqrt{K_{CI}}}{K_P} \frac{\Delta P_P \sqrt{\rho}}{\sqrt{\Delta P_{CI}}} \ (N\ s\ m^{-2}) \qquad (8)$$

where '$K_R$' is the ratio of coefficient of viscous pressure drop in the convergent to that in the parallel laminar fluid flow passage and is defined as:

$$K_R = \frac{K_{CV}}{K_P} \qquad (9)$$

and where $$\Delta P_{CI} = \Delta P_C - K_R\ \Delta P_P \qquad (10)$$

Herein it will be recognized that the inertia induced pressure drop '$\Delta P_{CI}$' cannot be measured directly but may be calculated when the two pressure drops '$\Delta P_C$' and '$\Delta P_P$' are measured and the ratio coefficient '$K_R$' is known.

Should a measure of the mass flow rate 'G' through the fluid passages be required, it may also be readily derived from equations (3) and (5) as:

$$G = \sqrt{\frac{\Delta P_{CI}\ \rho}{K_{CI}}} \ (kg\ s^{-1}) \qquad (11)$$

Accordingly, the practical implementation of an operational viscometer system is anticipated to consist, as per Fig. 3, of:

- a flow element containing two suitably-shaped laminar flow passages through which the Newtonian fluid of unknown viscosity continuously flows.

- a pressure transducer (or pair of transducers).

- a dedicated microprocessor.

- a display unit.

Figure 3 shows the schematic of a single pressure transducer viscometer system wherein a selector valve, actuated by the microprocessor, is used to intermittently switch the appropriate flow passage wall pressure tap line connections to the single transducer. The microprocessor utilizes the two pressure differential signals to calculate the fluid viscosity using the appropriate derived algorithms with the previously entered fluid density and geometric constants of the flow passages. The calculated fluid viscosity is subsequently displayed.

## 3.0 ASSESSMENT OF NON-DIMENSIONALIZED VISCOSITY ERROR STANDARD DEVIATION

In practice, the laminar flow passages must be designed so as to minimize the error associated with the fluid viscosity prediction subject to the accuracy limitations of commercially available transducers. The viscosity prediction error analysis is based on the following assumptions:

- the laminar flow passages are series-connected and exhibit no leakage such that the fluid mass through-flow rate 'G' is identical through each passage.

- the fluid density estimation error is assumed to exhibit a normal distribution (see Fig. 4) about a zero mean, provided that the initial estimate of fluid density is accurate. The assumption of a normal density error distribution is considered realistic in that small variations in fluid pressure, temperature and composition would be expected to occur randomly such that the dependent density error would exhibit a normal distribution.

- the fluid density estimation '$\rho$' has a maximum error of '$\pm\rho_e$' which is assumed to be three times the standard deviation '$\sigma$' of the normal error distribution on the basis that there is a 99.7% probability of the density error being within $\pm 3\sigma$ of the zero mean as per Fig. 4.

- the pressure measurement errors in the parallel and convergent passages are assumed to be independent and to exhibit normal distributions about a zero mean provided that transducer bias errors are effectively eliminated by appropriate calibration of the pressure transducers.

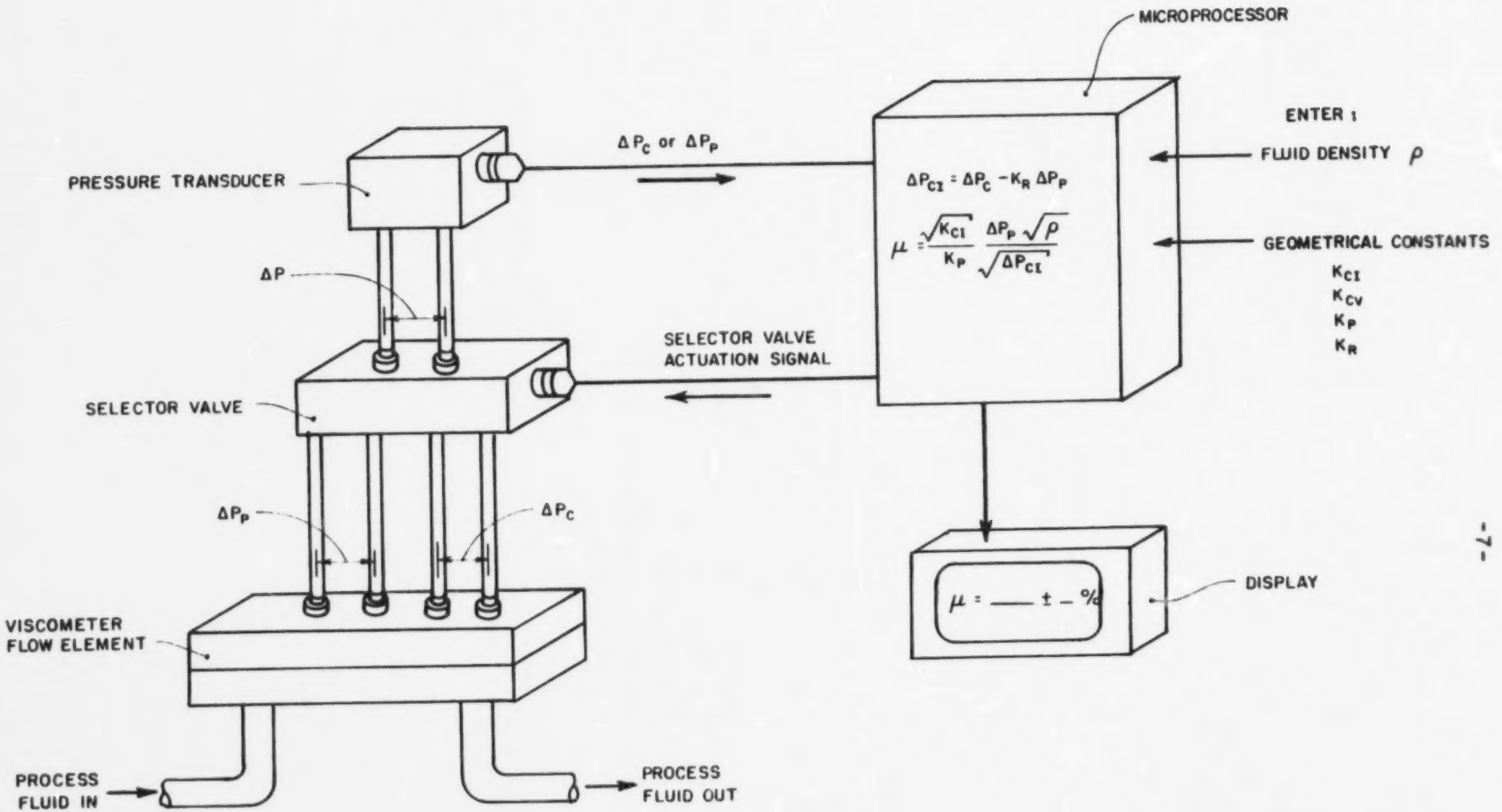
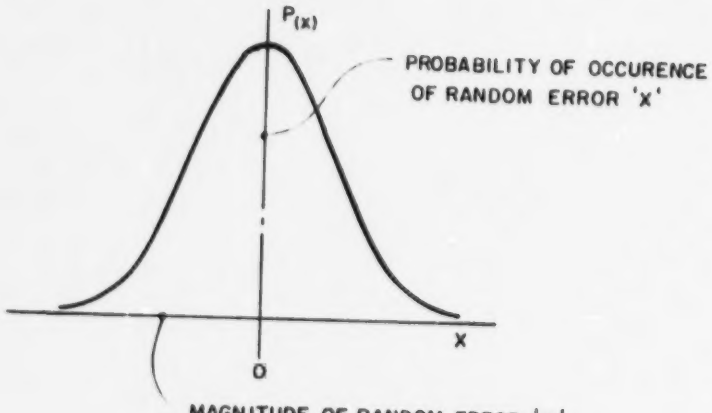



FIGURE 3

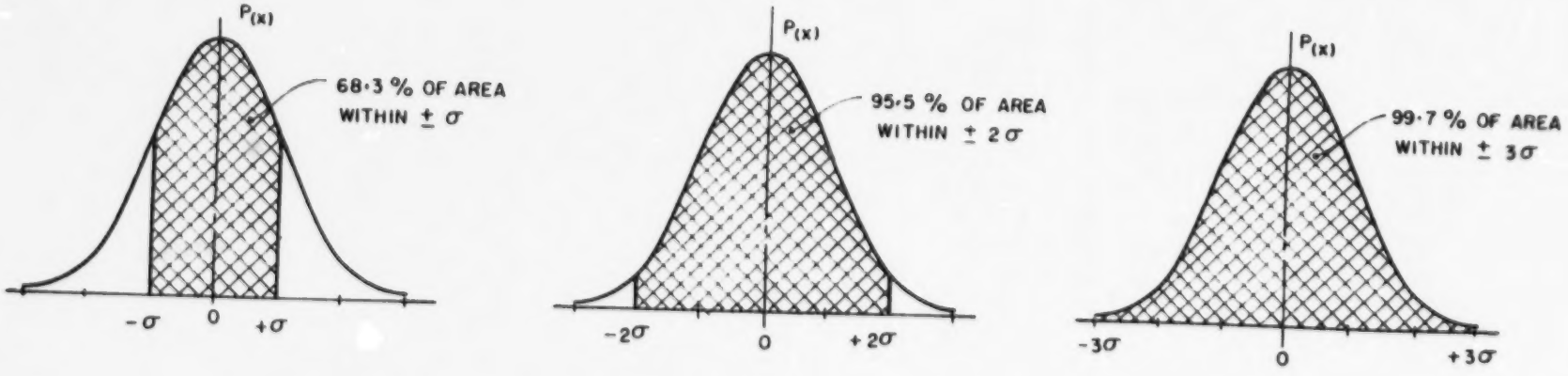
SCHEMATIC OF VISCOMETER SYSTEM USING SINGLE PRESSURE TRANSDUCER




FOR NORMAL ERROR DISTRIBUTION WITH ZERO MEAN

$$P_{(x)} = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{x^2}{2\sigma^2}}$$

WHERE $\sigma$ = STANDARD DEVIATION $= \sqrt{\overline{x^2}}$




AREA UNDER NORMAL DISTRIBUTION CURVE INDICATES PROBABILITY OF OCCURENCE WITHIN SELECTED RANGE

FIGURE 4

CHARACTERISTICS OF THE NORMAL OR GAUSSIAN ERROR DISTRIBUTION

- the fluid pressure drops in the convergent and parallel flow passages have maximum errors of $\pm p_{ec}$ and $\pm p_{ep}$ respectively where the maximum errors are assumed to be three times the standard deviation '$\sigma$' of the normal distribution on the basis that each pressure error has a 99.7% probability of being within $\pm 3\sigma$ of its zero mean as per Fig. 4.

The error of a sensor or transducer may be conveniently expressed in terms of the standard deviation of its distribution when such an error is dependent upon statistically distributed error sources as is the case of the fluid viscometer considered here. Further, the random viscosity prediction error of the viscometer will exhibit a normal distribution since the fluid density estimation and the pressure transducer measurement errors are independent of each other and each exhibit normal distributions and since any summation of normal distributions may be shown to exhibit a normal distribution (Bendat, J.S. and Piersol, A.G., Chap.3, 1966). Accordingly, the standard deviation of the normally distributed viscosity prediction error may be considered to represent a "weighted average" of its constituent errors and there will be a 68.3% probability of the viscometer error being within the '$\pm\sigma$' as per Fig. 4.

The standard deviation of the viscometer non-dimensionalized viscosity prediction error $\frac{\mu_e}{\mu}$ is shown (in Appendix A) to be related, in accordance with the appropriate statistical theorem for combining independent normal distributions, to the defined maximum fluid density estimation and pressure measurement errors by the expression:

$$\sigma = \frac{1}{3}\sqrt{\left(\frac{\rho_e}{2\rho}\right)^2 + \left|p_{ep}\left(\frac{1}{\Delta p_p} + \frac{K_R}{2\Delta P_{CI}}\right)\right|^2 + \left(\frac{p_{ec}}{2\Delta P_{CI}}\right)^2} \qquad (12)$$

## 4.0 PRACTICAL CONSIDERATIONS IN PASSAGE DESIGN

It is advantageous in practice to restrict both flow passages to designs for substantially two-dimensional rather than three-dimensional laminar fluid flow in that two-dimensional flow passages are more readily amenable to low cost precision manufacture, are less sensitive to localized surface damage, and are more serviceable with respect to disassembly for ease of surface inspection, cleaning, etc. Also, two-dimensional flow passages facilitate the accurate theoretical prediction of the viscometer performance on the basis of the relatively simple and extensive experimentally verified basic fluid dynamics for two-dimensional flow (Hayes, W.F. and Tucker, H.G, 1974, and Hayes, W.F., Tanney, J.W. and Tucker, H.G., 1974).

To ensure that fully developed two-dimensional laminar flow is maintained between the two pressure measurement locations within each flow passage, the following design criteria with respect to the passage geometry are recommended:

- the aspect ratio $\frac{'w_p'}{h_p}$ of the parallel flow passage cross-section should be at least ten such that the fluid flow approaches that for an infinitely wide passage where the effects of the viscous boundary layer associated with the side walls are negligible.

- in both the parallel and the convergent flow passages, the fluid flow Reynolds Number ($R_e$) is everywhere less than 7200 (the theoretical critical Reynolds Number for transition to turbulent flow for the two-dimensional parallel flow case), where

$$R_e = \frac{\rho V h}{\mu}$$

This latter criteria for the convergent flow passage is particularly conservative in that the favourable pressure gradient associated with the convergency can maintain laminar flow to substantially higher Reynolds Number (Hayes, W.F., Tanney, J.W. and Tucker, H.G., 1974).

- the upstream pressure tap, in both the parallel and convergent flow passages, is located such that the local laminar flow velocity profile is fully developed, i.e. at a distance from the passage inlet at least equal to the entrance length, '$l_e$', as given by Schlichting, H., 1968 for laminar flow in a parallel two-dimensional square-edged passage as:

$$l_e = 0.04\ R_e\ h$$

This latter criteria is particularly conservative in the case of the convergent passage where the favourable pressure gradient associated with the convergency tends to substantially reduce the passage length relative to that given above for the parallel passage case.

Further, there are the following fluid passage geometrical considerations associated with practical pressure measurement techniques and fluid power requirements:

- the pressure taps in the parallel passage should be displaced at least three times the passage height from the nearest adjacent side wall so as to eliminate any side wall boundary layer effects on the pressure measurement.

- at least two or three static pressure taps, manifolded together at each measurement location, should be used as shown in Fig. 1 and 2 in order to minimize probability of tap clogging.

- the laminar flow convergent passage radius ratio $r_2/r_1$ must be sufficiently small relative to the diameter of the wall static pressure taps so that the pressure gradient across the taps does not

introduce significant error into the pressure measurement. In contrast, there will be a constant pressure gradient along the parallel flow passage and thus pressure taps of the same dimension, located anywhere in the parallel passage, will be subjected to the same pressure gradient and accordingly will not introduce any tap size induced pressure drop measurement error.

- the inner radius '$r_2$' of the laminar flow convergent passage should be sufficiently large relative to the height '$h_c$' of the passage so that acceptably low fluid energy dissipation within the outlet pipe from the convergent passage is realized. As a design rule of thumb, the following convergent passage exhaust outlet design criteria have been found appropriate:

$r_2 > 2h_c$ for a double-sided fluid outlet pipe, (for example as per the viscometer design shown in Fig. 5).

and $r_2 > 4h_c$ for a single-sided fluid outlet pipe.

- the geometric parameters of the fluid flow passages must be selected such that a pressure drop compatible with commerically available transducers having acceptable accuracy can be realized.

- fluid passage height must exceed the maximum anticipated size of particulate matter suspended in the fluid.

## 5.0 VISCOMETER ERROR ASSESSMENT FOR TWO POSSIBLE DESIGN CONFIGURATIONS

### 5.1 Single Pressure Transducer Viscometer Design

It would appear advantageous to investigate the practicality of using a single pressure transducer appropriately connected via a suitable fluid line selector valve [as per Fig. 3], to intermittently measure the pressure drops across each flow passage for purposes of minimizing the viscometer hardware costs. Typically the accuracy characteristics of commercial differential pressure transducers may be realistically defined in terms of a fixed percentage of the transducer full scale reading. Accordingly, the full accuracy potential of such a transducer will be realized only by designing both of the viscometer laminar flow passages for pressure drops between the wall pressure taps which approach the full scale of the transducer. Thus the use of a single transducer requires that the pressure drops between the taps of each laminar flow passage are approximately equal whereby:

$$\Delta P_P \approx \Delta P_C = \Delta P$$

Also, such a single transducer implementation of the viscometer necessarily dictates that the maximum error in the pressure measurement for the parallel and convergent passages are identical whereby:

$$P_{ep} = P_{ec} = P_e$$

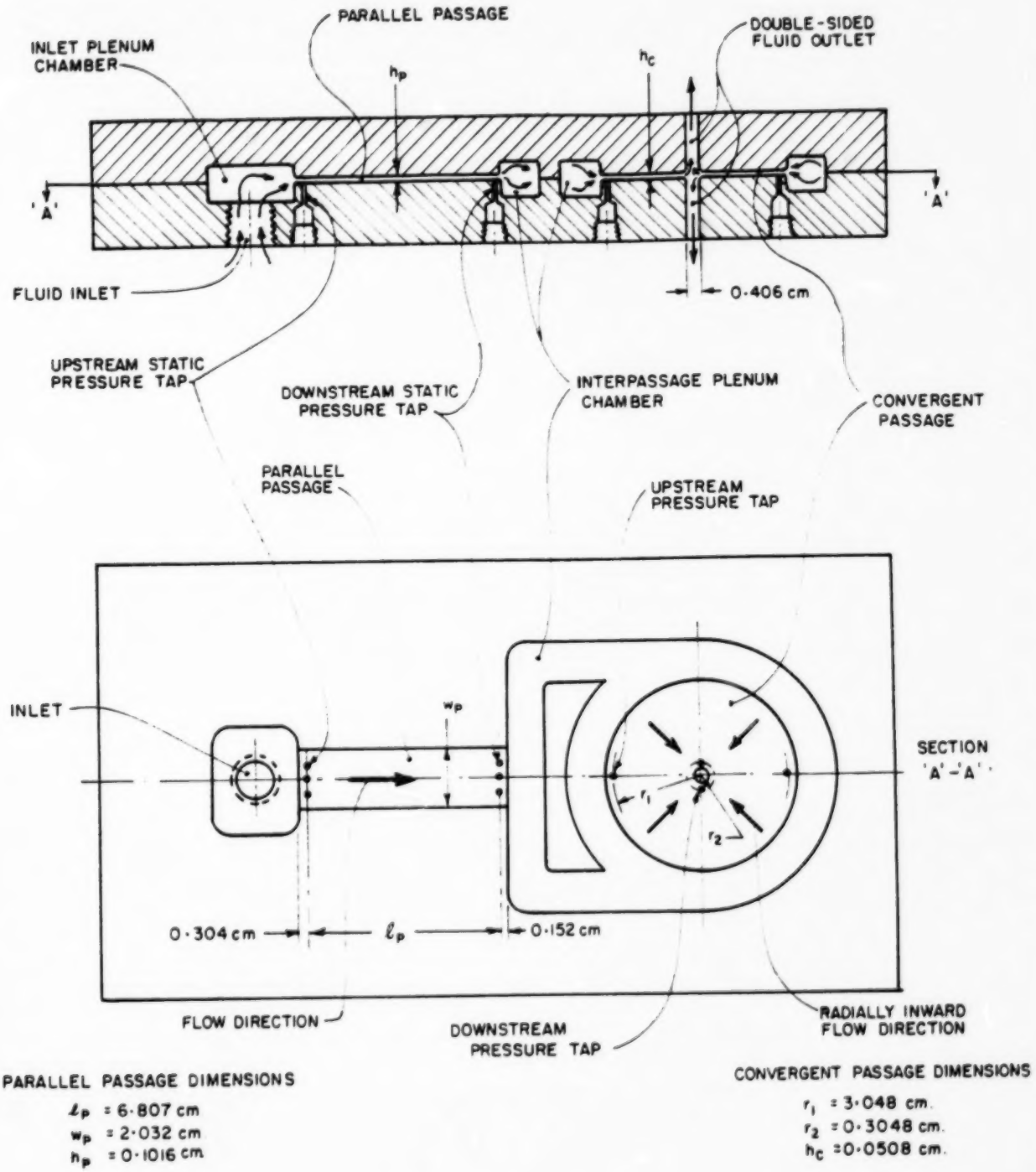
Accordingly, the standard deviation of the normalized viscosity error, as per Eqn. (12), reduces, for the single pressure transducer case, to:

$$\sigma_{ST} = \frac{1}{3}\sqrt{\left(\frac{P_e}{2\rho}\right)^2 + \left(\frac{P_e}{\Delta P}\right)^2 \left|1 + \frac{K_R}{1-K_R} + \frac{K_R^2+1}{4(1-K_R)^2}\right|} \qquad (13)$$

Only the $\left|1 + \frac{K_R}{1-K_R} + \frac{K_R^2+1}{4(1-K_R)^2}\right|$ term in Eqn. (13) is amenable to laminar flow passage design optimization. This '$K_R$' dependent term has been evaluated for the range $0 < K_R < 1$, as shown in Fig. 6, since it can be shown from Eqn. (10) that when $\Delta P_P = \Delta P_C$, $K_R$ must be less than unity. It will be evident from this figure that the viscosity prediction error is highly sensitive to the parameter '$K_R$' such that as $K_R$ approaches unity, the error approaches infinity. Further, it will be evident that the minimum viscosity prediction error will be realized when $K_R = K_{CV}/K_p = 0$ (i.e. when the viscosity-induced pressure drop in the convergent passage, $\frac{K_{CV}\,\mu G}{\rho}$, is zero). However, hardware implementation of the minimum error condition of zero viscosity-induced pressure drop in the convergent requires a turbulent flow orifice type device, with its attendant adverse features, as considered in Section 2. Accordingly, in order to minimize the viscometer error, '$\sigma_{ST}$', it is necessary to maximize the inertia pressure drop relative to the viscous pressure drop in the convergent passage by the appropriate choice of convergent passage geometric variables.

Considering only the geometry dependent variables in the convergent passage pressure drop expression (Eqn. (5)), the ratio of the inertia pressure drop to the viscous pressure drop may be shown to exhibit the proportionality:

$$\frac{\Delta P_{CI}}{\Delta P_{CV}} \propto \frac{h_c}{r_2^2}\left|1 - \left(\frac{r_2}{r_1}\right)^2\right| \frac{1}{\left|\ln \frac{r_2}{r_1}\right|} \qquad (14)$$




PARALLEL PASSAGE DIMENSIONS

$\ell_p$ = 6·807 cm
$w_p$ = 2·032 cm
$h_p$ = 0·1016 cm

CONVERGENT PASSAGE DIMENSIONS

$r_1$ = 3·048 cm.
$r_2$ = 0·3048 cm.
$h_c$ = 0·0508 cm.

FIGURE 5

CONFIGURATION OF 'BASELINE DESIGN' EXPERIMENTAL VISCOMETER SHOWING LAMINAR FLOW PASSAGE GEOMETRIES

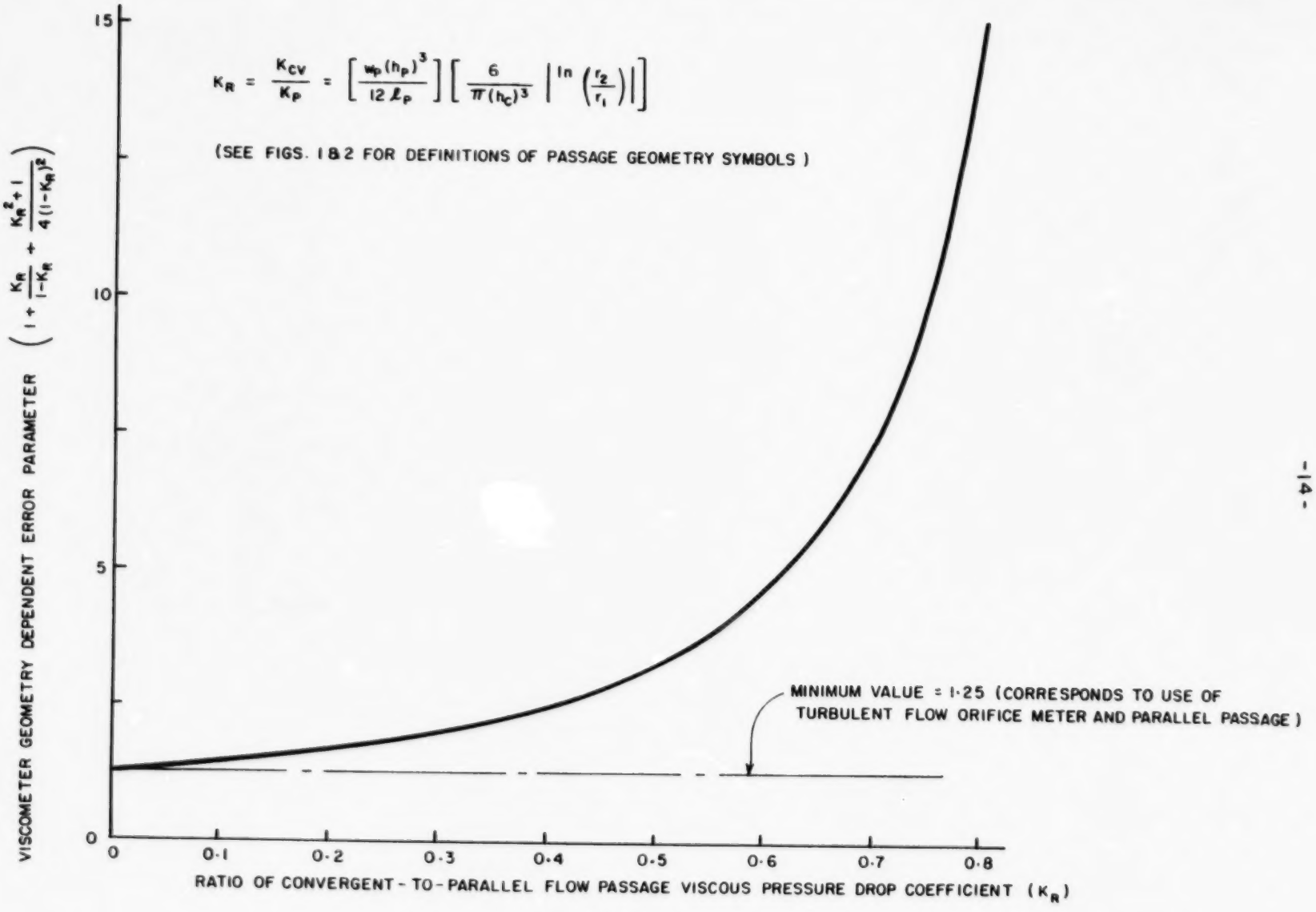



FIGURE 6

VISCOMETER ERROR DEPENDENCY UPON GEOMETRY OF LAMINAR FLOW PASSAGES
(SINGLE PRESSURE TRANSDUCER VISCOMETER CONFIGURATION)

Thus, in order to maximize the pressure drop ratio as defined by Eqn. (14) (i.e. to minimize the viscosity error as per Eqn. (13)), it is necessary, within the limiations of the practical design criteria of Section 4, to:

- maximize the convergent passage height, $h_c$;
- minimize the convergent inner radius, $r_2$;
- select a convergent passage radius ratio, $r_2/r_1$, which approaches unity.

There is the further requirement, as per Eqn. (13), that the parameter '$K_R$' be a minimum in order to minimize the viscosity prediction error. Herein, the parameter '$K_R$' may be shown, by combining Eqns. (4), (6) and (9), to exhibit the proportionality to the terms of the geometry dependent variables of both the parallel and the convergent laminar flow passages as given by:

$$K_R \propto \frac{h_p}{h_c}^3 \frac{w_p}{l_p} \left| \ln \frac{r_2}{r_1} \right| \qquad (15)$$

It will be evident from Eqn. (15) that '$K_R$' is highly sensitive to the parallel-to-convergent passage height ratio $h_p/h_c$, and moderately sensitive to the convergent passage radius ratio, $r_2/r_1$. Accordingly, in order to minimize $K_R$ it is necessary, within the limitations of the viscometer practical design criteria as per Section 4, to:

- minimize the height ratio $h_p/h_c$;
- select a convergent passage radius ratio $r_2/r_1$ which approaches unity;
- minimize the parallel passage width $w_p$;
- maximize the parallel passage length $l_p$.

The hardware implementation of such optimal fluid passage geometric conditions for minimal viscometer error corresponds in the limit to a very long laminar flow capillary tube having minimum flow passage cross-section connected in series with an orifice plate which inherently implies turbulent flow. However, for the reasons previously considered in Section 2.0, the use of a turbulent flow device is not considered practical for the repeatable long-term performance required for a precision viscometer.

The introduction of the fluid passage defining geometric parameters of the pressure drop ratio, as per Eqn. (14), or of the coefficient $K_R$, as per Eqn. (15), into the viscosity error Eqn. (13) results in an expression which is complicated to the extent that trends of the error dependency upon individual or groups of geometric parameters are not directly evident without explicit numerical evaluations of the complete equation for specific design cases. Accordingly, it is appropriate to numerically evaluate the error equation in terms of particular selected geometric parameters in accordance with the limitations as per Section 4.0 or any other limitations as might apply in any particular application. In order to investigate the practical design tradeoffs between fluid passage geometry, pressure measurement error, density estimation error and overall sensor performance, an iterative design procedure using the derived expressions for the viscometer error is required.

An experimental viscometer, incorporating a typical commercial variable reluctance type pressure transducer having a full scale differential pressure measurement capability of approximately $1.14 \times 10^5$ N $m^{-2}$ and a maximum pressure measurement error of approximately $\pm$ 1705 N $m^{-2}$ or $\pm$ 1.5% of full-scale, was designed and fabricated to measure the viscosity of a particular hydraulic oil (identified as Tellus 33) having a nominal viscosity of 0.060 N s $m^{-2}$ at 24°C and a nominal density of 920 kg $m^{-3}$.

The passage dimensions were optimized for a nominal fluid flow $4.5 \times 10^{-2}$ kg $s^{-1}$, within practical sizing and fabrication tolerance limitations, as well as within the limitations of the practical design criteria of Section 4, to give the following viscometer "baseline" design geometry:

$$l_p = 6.807 \text{ cm}$$

$$w_p = 2.032 \text{ cm}$$

$$h_p = 0.1016 \text{ cm}$$

$$r_2 = 0.3048 \text{ cm}$$

$$r_1 = 3.048 \text{ cm}$$

$$h_c = 0.0508 \text{ cm}$$

The prototype test hardware for this "baseline" design viscometer, having the geometry of laminar flow passages shown in Fig. 5, is shown disassembled in Fig. 7 and under test in an experimental fluid flow rig in Fig. 8. An error analysis for this particular viscometer configuration resulted in a non-dimensionalized viscosity error standard deviation of approximately 3% (i.e. a 68.3% probability that the predicted viscosity would be within ± 3% of the correct value based on a normal distribution of error as per Fig. 4).

The effects of variations in flow rate or in fluid viscosity on the viscosity prediction error for this "baseline" viscometer design may be readily determined by evaluation of Eqn. (12), and are shown in Fig. 9. Herein, it is appropriate to use the generalized expression for viscosity error as per Eqn. (12) rather than the simplified Eqn. (13) since off the design point the pressure drops across the two laminar flow passages will not be exactly equal. However, the magnitude of maximum transducer pressure measurement error will remain equal to the error magnitude at the nominal design condition (i.e. at 1.5% of the design point pressure drop or 1705 N $m^{-2}$), since the pressure transducer hardware will remain unaltered. It will be evident from Fig. 9 that, although substantial variations in viscosity do not significantly change the viscosity estimation, variations in flow rate will significantly affect the error shown. Also shown in Fig. 9 is the viscosity error reduction achievable when the maximum errors in pressure measurement and density estimation are reduced from 1.5% to 1.0% and to 0.5%.

It will be shown in the following section (5.2) that any cost advantage associated with using a single transducer and associated fluid line selector valve is likely to be more than offset by the degradation in viscometer performance as induced by the requirement for nearly equal pressure drops in the two flow passages.

### 5.2 Dual Pressure Transducer Viscometer Design

The practical viscometer design tradeoffs between fluid passage geometry, pressure measurement error, density estimation error and overall sensor performance are determined by numerically evaluating examples of viscometer geometry and fluid flow conditions. Accordingly, when considering implementation of the viscometer with two pressure transducers, such that the two laminar flow passages may be designed to have significantly different pressure drops, it is convenient for performance comparison purposes to use the "baseline design point" (as per Section 5.1) as the reference condition when illustrating the effect on the viscosity estimation error of varying particular viscometer parameters. Herein it has been assumed for the error prediction of the dual transducer viscometer that:

- the parallel passage geometry is fixed at its "baseline design condition" and the magnitude of the maximum pressure measurement error in the parallel passage $\pm p_{ep}$ remains constant at 1.5% of the "design point" pressure drop.

- the height and radius ratio of the convergent passage are independently varied from their baseline design point values.

- the full scale range of the convergent passage pressure transducer is presumed to vary so as to be substantially equal to the passage pressure drop such that the magnitude of the maximum pressure measurement error in the convergent passage '$\pm p_{ec}$' is always 1.5% of the actual rather than the "design point" pressure drop (corresponds to always selecting a pressure transducer having a full scale range matched to the actual pressure drop of the convergent flow passage).

It may be readily shown, from an appropriate fluid flow passage geometry dependent error sensitivity analysis for the selected convergent passage geometry, that:

- increasing the height, $h_c$, of the convergent passage will substantially reduce the viscosity estimation error (to less than 2% at $h_c$ = 0.10 cm) while also reducing the convergent passage pressure drop as shown in Fig. 10. Herein, the maximum convergent passage height is limited by the corresponding minimum pressure which is measurable in practice by commercially available pressure transducers. Also shown in Fig. 10 is the reduction in viscosity error achievable when the maximum errors in pressure measurement and density estimation are reduced from 1.5% to 1.0% and to 0.5%.

- increasing the radius ratio, $r_2/r_1$, of the convergent passage will reduce both the viscosity estimation error (to about 1.5% at $r_2/r_1$ = 0.6) and the convergent passage pressure drop as shown in Fig. 11. Herein, the maximum radius ratio is limited in practice to something less than unity by the corresponding passage minimum pressure drop which is measurable in practice by commercially available pressure transducers. Also shown in Fig. 11 is the reduction in viscosity error achievable when the maximum errors in pressure measurement and density estimation are reduced from 1.5% to 1.0% and to 0.5%.

It will be evident from Fig. 10 and 11 that the performance of a dual pressure transducer viscometer can be significantly improved relative to that of the single pressure transducer "baseline" viscometer, by increasing the convergent passage height, $h_c$, and/or radius ratio, $r_2/r_1$, when the density estimation and transducer errors are equivalent.

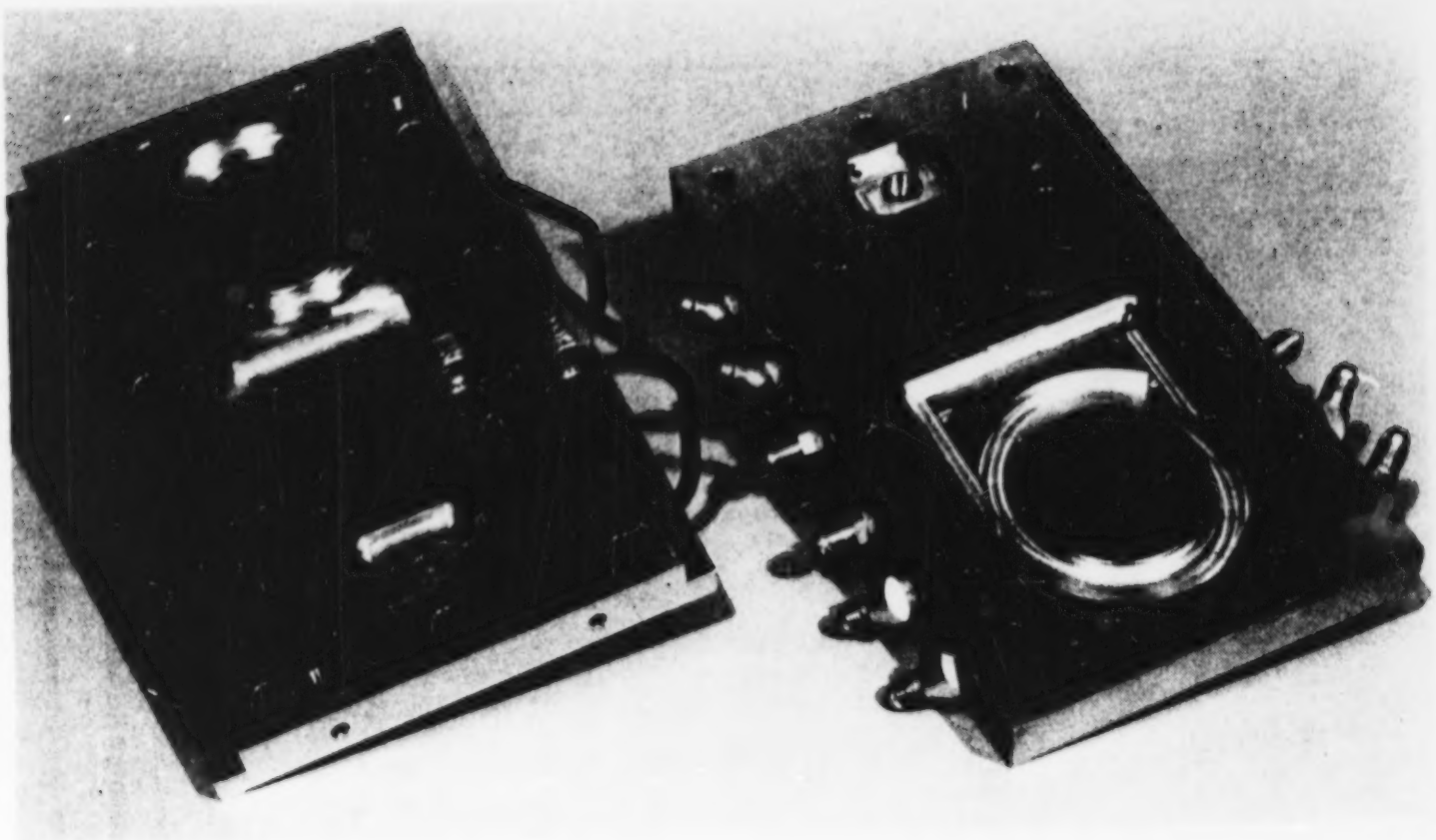




FIGURE 7

DISASSEMBLED 'BASELINE DESIGN' VISCOMETER FLOW ELEMENT

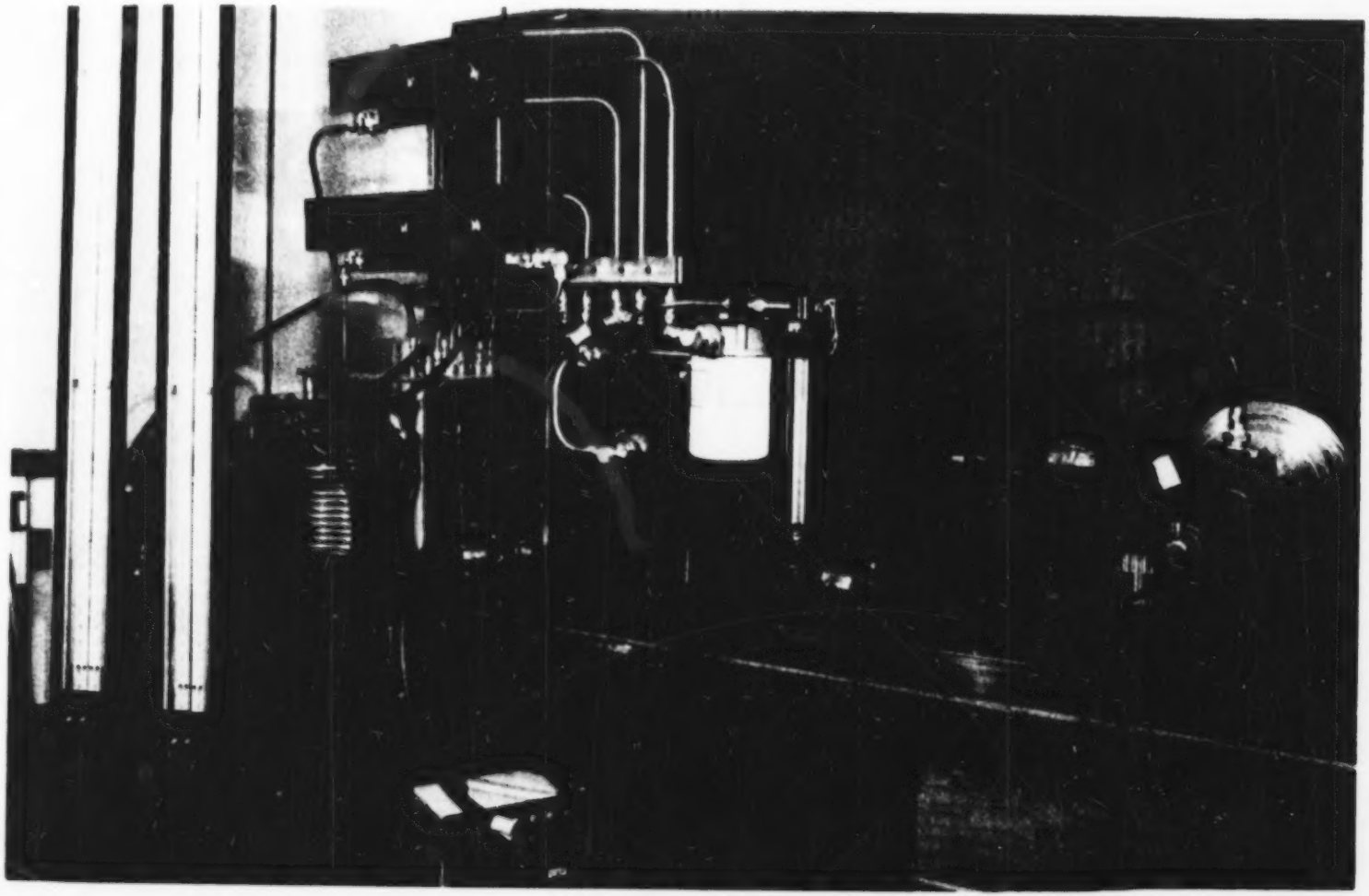




FIGURE 8

EXPERIMENTAL FLOW TEST RIG FOR VISCOMETER FLOW ELEMENT

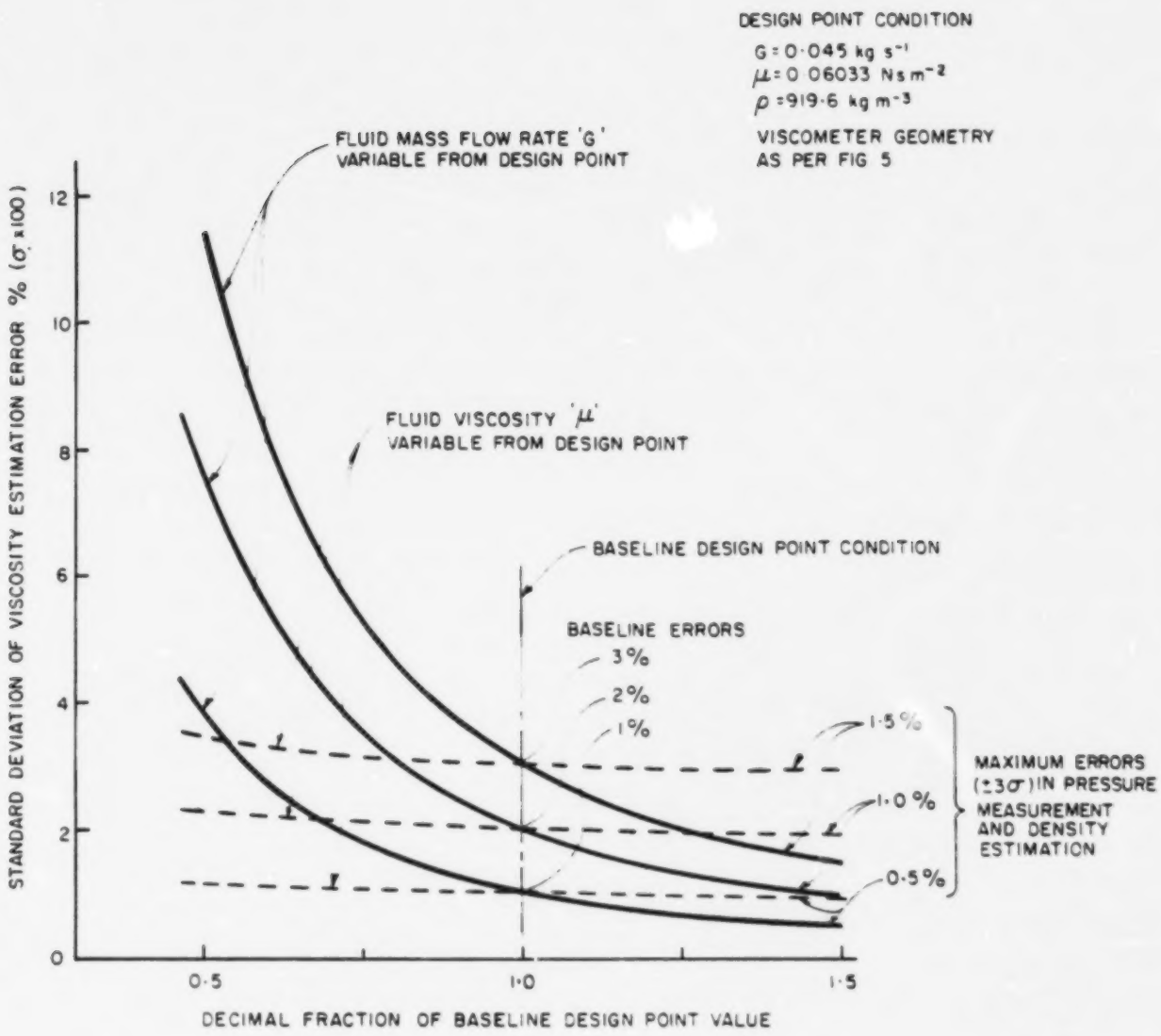
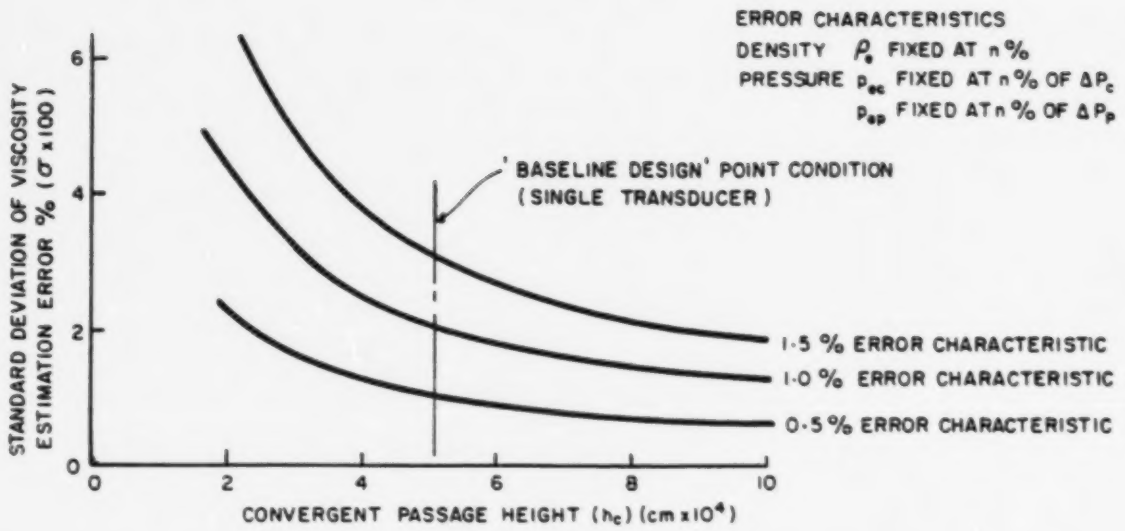



FIGURE 9

VARIATION OF VISCOSITY ERROR WITH DEVIATION OF FLUID FLOW RATE OR VISCOSITY FROM DESIGN POINT. (SINGLE PRESSURE TRANSDUCER VISCOMETER CONFIGURATION)


ERROR CHARACTERISTICS
DENSITY $\rho_e$ FIXED AT n %
PRESSURE $p_{ec}$ FIXED AT n % OF $\Delta P_c$
$p_{ep}$ FIXED AT n % OF $\Delta P_p$
'BASELINE DESIGN' POINT CONDITION
(SINGLE TRANSDUCER)
1·5 % ERROR CHARACTERISTIC
1·0 % ERROR CHARACTERISTIC
0·5 % ERROR CHARACTERISTIC
STANDARD DEVIATION OF VISCOSITY ESTIMATION ERROR % ($\sigma$ x100)
CONVERGENT PASSAGE HEIGHT ($h_c$) (cm x $10^4$)

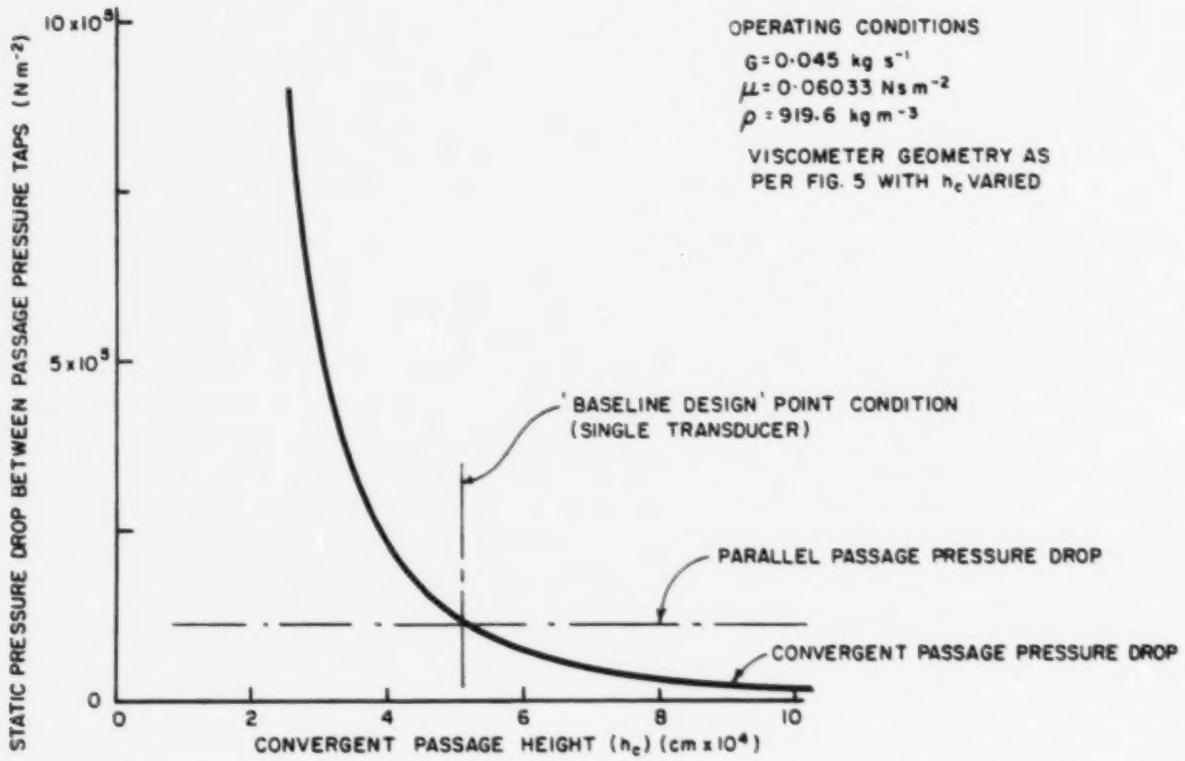
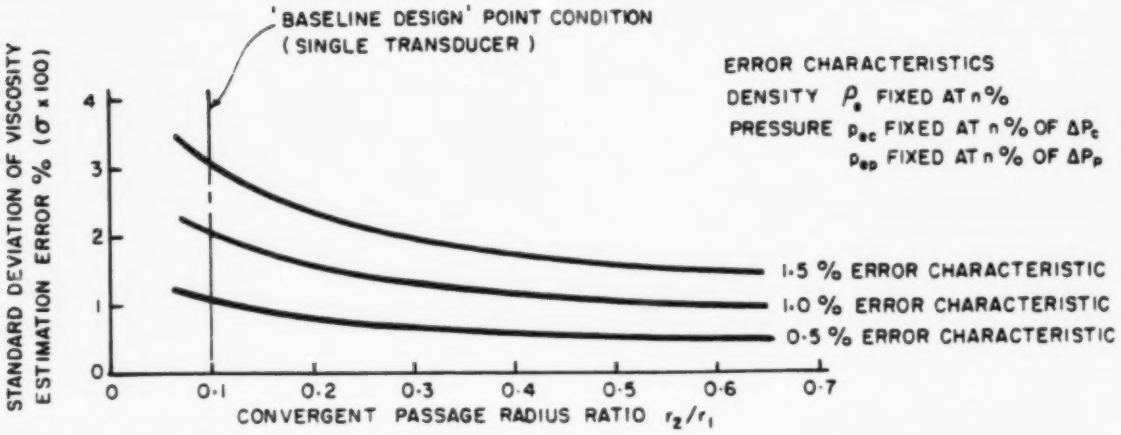

OPERATING CONDITIONS
G = 0·045 kg $s^{-1}$
$\mu$ = 0·06033 Ns $m^{-2}$
$\rho$ = 919·6 kg $m^{-3}$
VISCOMETER GEOMETRY AS PER FIG. 5 WITH $h_c$ VARIED
'BASELINE DESIGN' POINT CONDITION
(SINGLE TRANSDUCER)
PARALLEL PASSAGE PRESSURE DROP
CONVERGENT PASSAGE PRESSURE DROP
STATIC PRESSURE DROP BETWEEN PASSAGE PRESSURE TAPS (N $m^{-2}$)
CONVERGENT PASSAGE HEIGHT ($h_c$) (cm x $10^4$)

FIGURE 10

VARIATION OF VISCOSITY ERROR AND FLOW PASSAGE PRESSURE DROPS WITH CONVERGENT PASSAGE HEIGHT (DUAL PRESSURE TRANSDUCER VISCOMETER CONFIGURATION)




OPERATING CONDITIONS
$G = 0{\cdot}045\ kg\ s^{-1}$
$\mu = 0{\cdot}06033\ N\,s\,m^{-2}$
$\rho = 919{\cdot}6\ kg\ m^{-3}$
VISCOMETER GEOMETRY AS PER FIG. 5 WITH $r_1$ VARIED

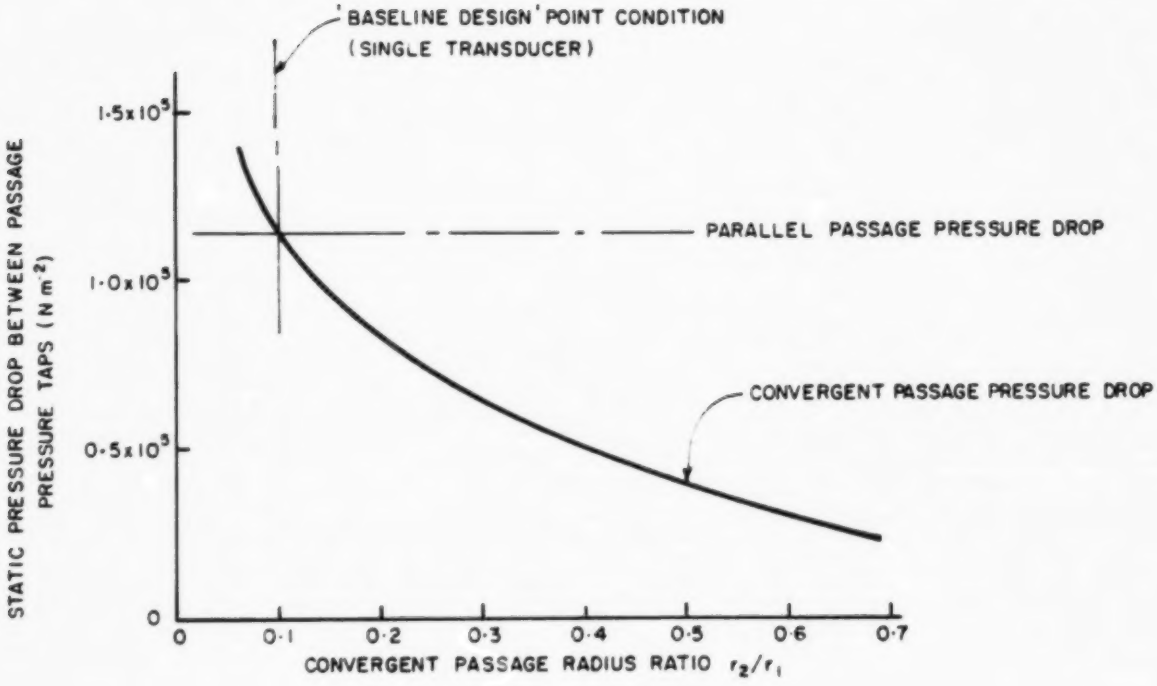



FIGURE 11

VARIATION OF VISCOSITY ERROR AND FLOW PASSAGE PRESSURE DROPS WITH CONVERGENT PASSAGE RADIUS RATIO. (DUAL PRESSURE TRANSDUCER VISCOMETER CONFIGURATION)

It can also be shown from an appropriate error sensitivity analysis of the selected "baseline" viscometer design that, when fluid density is estimated to within a maximum of ± 1.5% of its nominal value and the pressure drops are measured to within a maximum of ± 1.5% of their nominal value, further unlimited increases in convergent passage height and radius ratio will yield a theoretical absolute minimum possible viscosity error standard deviation of 0.6%. It is obvious that the actual viscosity error which is achievable in practice will always be somewhat greater than the theoretical minimum due to the practical limitations of physical size and minimum detectable pressure drop.

## 6.0 CONCLUSIONS

The viscosity estimation error of the patented viscometer is considered to be well within commercially acceptable limits when the viscometer performance is based on typical differential pressure transducer accuracy and anticipated realistic fluid density drift. This viscosity error is not dependent upon the fundamental fluid flow process of the viscometer but rather is dependent upon the errors in the measurement of the pressure drop between the wall taps of the viscometer laminar flow passages and the error in the estimation of fluid density. In practice, the minimum achievable viscosity error is necessarily a design tradeoff between:

- practical viscometer fluid flow passage sizing and fabrication tolerance limitations.
- geometrical limitations imposed by the requirement that laminar flow be maintained in both fluid flow passages.
- geometrical limitations associated with practical pressure transducer measurement considerations and fluid power requirements.

Any cost advantage associated with using a single transducer and associated fluid line selector valve, would be expected to be more than offset by the degradation in viscometer performance, which is shown by the viscometer error analysis to be induced by designing the two flow passages to have approximately equal pressure drops.

It is not practical to obtain a general expression for the dependency of the trends of viscosity error upon the various specific geometric parameters of the viscometer laminar flow passages. However, the viscosity error can be readily predicted for any specific viscometer design, when the appropriate fluid flow parameters are known, by numerical evaluation of the derived error equations.

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## APPENDIX A
## DERIVATION OF NON-DIMENSIONALIZED VISCOSITY ERROR STANDARD DEVIATION

The error, $\mu_e$, in the viscometer predicted viscosity '$\mu$' as induced by the maximum errors $p_{ep}$ and $p_{ec}$ in the pressure measurements $\Delta P_P$ and $\Delta P_C$ respectively and the maximum error, $\rho_e$, in the density estimation, $\rho$, may be expressed, by substituting these errors into the viscosity determination Eqn. (8) as:

$$\mu+(\pm\mu_e) = \sqrt{\frac{K_{CI}}{K_P}} \frac{\sqrt{\rho + (\pm\rho_e)}\,[\Delta P_P + (\pm p_{ep})]}{\sqrt{[\Delta P_C + (\pm p_{ec})] - K_R\,[\Delta P_P + (\pm p_{ep})]}} \qquad (A\text{-}1)$$

Herein, the denominator may be rewritten and series expanded as

$$\sqrt{\frac{1}{\Delta P_{CI}+(\pm p_{ec})-K_R(\pm p_{ep})}} = \sqrt{\frac{1}{\Delta P_{CI}} - \frac{[(\pm p_{ec})-K_R(\pm p_{ep})]}{(\Delta P_{CI})^2} + \frac{[(\pm p_{ec})-K_R(\pm p_{ep})]^2}{(\Delta P_{CI})^3} - \ldots}$$

$$\simeq \sqrt{\frac{1}{\Delta P_{CI}} - \frac{[(\pm p_{ec})-K_R(\pm p_{ep})]}{(\Delta P_{CI})^2}} \qquad (A\text{-}2)$$

since all subsequent terms consist of second or higher order error terms and may be neglected.

Substituting Eqn. (A-2) into Eqn. (A-1), squaring and ignoring second and higher order error terms yields:

$$\mu^2+2(\pm\mu_e)\mu = \frac{K_{CI}}{(K_P)^2}\frac{1}{(\Delta P_{CI})}\,[\rho(\Delta p_p)^2 + 2(\pm p_{ep})(\Delta p_p)\rho + (\pm\rho_e)(\Delta p_p)^2]$$

$$- \frac{K_{CI}}{(K_P)^2}\,\frac{1}{(\Delta P_{CI})^2}\,\{\rho(\Delta p_p)^2\,[(\pm p_{ec})-K_R(\pm p_{ep})]\} \qquad (A\text{-}3)$$

Subtracting $\mu^2$, as derived from Eqn. (8), from both sides of Eqn (A-3) and dividing the resulting equation by $2\mu^2$ results in the non-dimensionalized viscosity error expression:

$$\frac{(\pm\mu_e)}{\mu} = \frac{\pm\rho_e}{2\rho} + (\pm p_{ep})\left|\frac{1}{\Delta P_P} + \frac{K_R}{2\Delta P_{CI}}\right| - \left|\frac{(\pm p_{ec})}{2\Delta P_{CI}}\right| \qquad (A\text{-}4)$$

The retention of the dual (±) signs implies that the transducer-induced errors in pressure measurement are independent from each other and from the density estimation error (i.e. any error may occur in either directional sign sense, independent of the other errors). Further, as per Section 3.0, each error has been assumed to exhibit a normal distribution such that each maximum error magnitude corresponds to three standard deviations. It may be theoretically shown (Schlichting, 1968, Chap. 5) that, for any number of normally distributed independent random variables having zero mean, the standard deviation of their summation is equal to the square root of the sum of the squares of the individual standard deviations. Thus, the standard deviation of the non-dimensionalized viscosity error may be expressed as:

$$\sigma = \frac{1}{3}\sqrt{\left(\frac{\rho_e}{2\rho}\right)^2 + P_{ep}\left|\frac{1}{\Delta P_P} + \frac{K_R}{2\Delta P_{CI}}\right|^2 + \left(\frac{P_{ec}}{2\Delta P_{CI}}\right)^2} \qquad \text{(A-5)}$$

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AUTHOR(S)/AUTEUR(S)

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NOTES

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viscometer, viscosity, laminar flow, error analysis

14

SUMMARY/SOMMAIRE

The report assesses the effects of errors in pressure measurement and density estimation upon the precision to which fluid viscosity may be predicted using a patented laminar flow viscometer.

15

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